Neutrino Oscillations by Interaction with Moduli

K. Kobayakawa[1] [*], Y. Sato[2] and S. Tanaka[3]

[1] *Fukui University of Technology,*
Gakuen-cho, Fukui 910, Japan

[2]*Graduate School of Science and Technology,*
Kobe University, Nada, Kobe 657, Japan

[3] *Division of Natural Environment and High Energy Physics,*
Faculty of Human Development,
Kobe University, Nada,Kobe 657, Japan

Abstract

We would like to point out the possibility to detect the low-energy signals of moduli in the superstring theory in the neutrino oscillation. The idea is based on the characteristics that the couplings of moduli are different in matter. We estimate the oscillation probability both in the long baseline and in the solar neutrino oscillations and examine the detectable region of the moduli effect.

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[*]On leave from Faculty of Human Development, Kobe University

Recent LEP data [1] suggest the evidence of Grand Unified Theories such as SU(5), SO(10), flipped SU(5) and so on. Furthermore the data fit better on including supersymmetry. On the theoretical side to solve the gauge hierarchy problem the idea of supersymmetry is very persuasive. However, SUSY GUT does not contain the interaction of gravity. At present it is conceived that the superstring theory alone may include all interactions consistently in the theory. Phenomenologically the heterotic superstring theory [2] is most attractive. There are several ways of compactification and after that there come out very many vacua [3]. They are parametrized, in general, by moduli [4] which are singlet superfields under the gauge group of the standard model, $SU(3)_{\mathrm{C}} \times SU(2)_{\mathrm{L}} \times U(1)_{\mathrm{Y}}$. For example, some of them describe the size and shape of the compactified space. But masses of moduli are not known, though their vacuum expectation values are supposed to be of the order of Planck scale. Their interactions with matter are also model-dependent. Even the number of moduli depend on the structure of the vacuum considered. Consequently it is very helpful to detect the moduli.

In this paper we would like to point out that moduli characteristic in the superstring theory may give new low-energy signals which could be tested in the neutrino oscillation experiments. Moduli generally couple to ordinary matter with nonrenormalizable interactions. Such couplings are expressed in the superpotential effectively as (in the lowest dimension)

$$P_{nonren} = \frac{c_{ijk}^I}{M_{\mathrm{S}}} \varphi_i \varphi_j \varphi_k M_I, \qquad (I = 1, 2, 3, \ldots), \qquad (1)$$

where $\varphi_{i,j,k}$ are matter superfields, M_I are moduli superfields and M_{S} is the string scale ($\sim 10^{18}$GeV). Such terms at low energies induce Yukawa-type couplings between the ordinary matter and (real) scalar fields or pseudoscalar fields i.e. moduli:

$$\begin{aligned}
\mathcal{L}_Y &= \frac{<H_2>}{M_{\mathrm{S}}} h_{ij}^{(\nu)} \bar{\nu}_R^i \nu_L^j M_I + \frac{<H_2>}{M_{\mathrm{S}}} h_{ij}^{(u)} \bar{u}_R^i u_L^j M_I \\
&+ \frac{<H_1>}{M_{\mathrm{S}}} h_{ij}^{(d)} \bar{d}_R^i d_L^j M_I + \frac{<H_1>}{M_{\mathrm{S}}} h_{ij}^{(\ell)} \bar{\ell}_R^i \ell_L^j M_I + h.c., \qquad (2)
\end{aligned}$$

where i and j are generation indices($i = 1, 2, 3$) and $<H_{1,2}>$ are the vacuum expectation values of the Higgs doublets and γ-matrices are dropped. While dilaton S interacts with ordinary matter universally like graviton, moduli interact (or not interact) with various couplings. Moduli interact with ordinary matter as a coherent attractive force [5]. (We also consider the possibility

of moduli interaction generating a repulsive force.) Since the interaction strength is comparable to that of gravity force, this behaves as a kind of fifth force if the mass of the exchanged particle is small enough [5][6]. Ordinary moduli mass is expected to be of the order of the gravitino mass. Because moduli have a flat potential perturbatively and they must get mass by non-perturbative effects. It is usually said that it occurs after supersymmetry breaking. So it would be as heavy as other scalar sparticles. However, there are arguments that some moduli would have very tiny mass:

(1) for real M_I the moduli mass (m_{M_I}) may be induced by radiative corrections ($m_{M_I} \simeq 10^{-18}$GeV) [5], or there may be a special cancellation of two terms in the mass equation [7]. In ref.[8], it is estimated that m_{M_I} can be about $m_{\frac{3}{2}}^2/ReM_I$, where $m_{\frac{3}{2}}$ is the gravitino mass.

(2) for imaginary M_I, in ref.[6] it is argued that m_{M_I} can be 2×10^{-24}GeV. However, in ref.[8] it is said that they are massless. In ref.[9], on the other hand, they are said to gain huge mass of the order of the SUSY-breaking scale.

There is no definite mass which can be calculated numerically, and also the form of the scalar potential is not known yet. So we do not get into the details of the models here and take a mass of a modulus (especially tiny one) as a parameter m_{M_I} and its relative interaction strength as parameters f_{ij}, and explore the possibilities of finding the effects of moduli in the terrestrial experiments, not in cosmology.

First we discuss long baseline neutrino oscillations. The planning experiments such as from FNAL to SOUDAN2 [10] is illustrated schematically in Fig.1. The neutrino source from an accelerator is located at the point x_1, and the detector at the point x_2. The muon neutrino (ν_μ) beam with energy E (of the order from one GeV to a few ten GeV) propagates along the x-axis. We assume, for simplicity, that there is at least one modulus which interacts with ν_τ and/or ν_μ and u- or d-quark (or electron). For example, $h_{22}^{(\nu)} \neq 0, h_{11}^{(u)} \neq 0$ and others can be zero in eq.(2). Although the interaction strength is gravitational, it may be detectable in the neutrino oscillations when m_{M_I} is very tiny. We take it in this paper in the range of $10^{-24} - 10^{-15}$GeV.

We define the eigenstate of mass plus moduli interaction as (ν_2, ν_3), and the flavor eigenstate as (ν_μ, ν_τ). The latter eigenstate is expressed by the

former with a mixing angle ζ as

$$\begin{pmatrix} \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} \cos\zeta & \sin\zeta \\ -\sin\zeta & \cos\zeta \end{pmatrix} \begin{pmatrix} \nu_2 \\ \nu_3 \end{pmatrix}. \tag{3}$$

The Hamiltonian of mass eigenstate is given by

$$H = \begin{pmatrix} p + \frac{m_2^2}{2p} - f_{22}\phi & -f_{23}\phi \\ -f_{32}\phi & p + \frac{m_3^2}{2p} - f_{33}\phi \end{pmatrix}, \tag{4}$$

where p is the momentum of a neutrino beam and m_2 and m_3 are the masses of mass eigenstates. $f_{ij}\phi$ in eq.(4) represent the potentials induced by moduli interaction. The difference of modulus coupling with matter is stuffed into $f_{ij}(|f_{ij}| \le 1; i, j = 2, 3)$. We can take $f_{23} = f_{32}$. Since f_{23} is not necessarily zero, this Hamiltonian is not always diagonal. In a relativistic case ϕ is represented as the product [11] of energy of a neutrino beam and the potential per unit mass due to moduli interaction with matter

$$\begin{aligned} \phi &= EV, \\ V &= G_M \frac{M}{r} \exp(-\mu r). \end{aligned} \tag{5}$$

Here μ can be regarded as almost same as the moduli mass and G_M is a common coupling constant of moduli so that the maximum value among $|f_{ij}|$ is unity. In eq.(5), M is the mass of the matter which is interacting with neutrino interchanging moduli.

To estimate V, the contribution to V from the whole earth is added up. Then

$$\begin{aligned} \phi_{global} &= -\frac{3G_M M_E E}{2\mu^2 R_E^3} \frac{R_E + \mu^{-1}}{\xi_0} \\ &\times [\exp\{-\mu(R_E - \xi_0)\} - \exp\{-\mu(R_E + \xi_0)\}] + \frac{3G_M M_E E}{\mu^2 R_E^3}, \end{aligned} \tag{6}$$

where M_E and R_E denote the mass and the radius of the earth, respectively. ξ_0 is the distance between the beam and the center of the earth as shown in Fig.1. We can put $\xi_0 = R_E$ approximately. The value of eq.(6) is almost the same as that of the sphere with radius μ^{-1} because of the exponential decrement in eq.(5).

The Hamiltonian (4) can be diagonalized and we get

$$i\frac{d}{dx}\begin{pmatrix} \nu_2 \\ \nu_3 \end{pmatrix} = \begin{pmatrix} p + \frac{m_2^2}{2p} + \alpha & 0 \\ 0 & p + \frac{m_3^2}{2p} + \beta \end{pmatrix} \begin{pmatrix} \nu_2 \\ \nu_3 \end{pmatrix}, \tag{7}$$

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where α and β are parameters introduced for convenience sake:

$$\alpha = \lambda_2 - p - \frac{m_2^2}{2p}, \tag{8}$$

$$\beta = \lambda_3 - p - \frac{m_3^2}{2p}, \tag{9}$$

$$\lambda_{2,3} = p + \frac{m_2^2 + m_3^2}{4p} - \frac{f_{22} + f_{33}}{2}\phi$$
$$\pm \left[\left(\frac{\Delta m^2}{4p}\right)^2 + \left(\frac{\Delta f \phi}{2}\right)^2 - \frac{\Delta m^2}{4p}\Delta f\phi + f_{23}^2\phi^2\right]^2, \tag{10}$$

where $\Delta m^2 = m_3^2 - m_2^2$ and $\Delta f = f_{33} - f_{22}(|\Delta f| \leq 1)$ represents the difference of the coupling constants of the two neutrino species with matter.

Solving eq.(7) and using eqs.(3) and (8)-(10), we obtain the oscillation probability:

$$P(\nu_\mu \rightarrow \nu_\tau)$$
$$= \sin^2 2\zeta$$
$$\times \sin^2\left[\frac{L}{c}\left\{\left(\frac{\Delta m^2}{4E}\right)^2 + \left(\frac{\Delta f \phi}{2}\right)^2 - \frac{\Delta m^2}{4E}\Delta f\phi + f_{23}^2\phi^2\right\}^{\frac{1}{2}}\right]. \tag{11}$$

The first term inside the brace is due to the vacuum oscillation and the last three terms are due to moduli interaction. So comparing the two kinds of contributions, we can examine the effect of moduli interaction. The former is proportional to E^{-1} and the latter is proportional to E. Therefore the higher the energy of neutrino is, the larger the effect of moduli is. The effect of moduli interaction may be detected experimentally, if it is at least about 10^{-3} of that of vacuum oscillation [12]. Here we neglect the term of $f_{23}^2\phi^2$ so it might be underestimation of moduli effect.

We examine the detectable region of two parameters μ and Δf as follows. The force induced by moduli interactions behaves like the fifth force which many experiments have tested and put restrictions. First fixing the value of μ, which we set at a reciprocal of the force range λ, we take G_M in eq.(5) at the maximum value of allowable G_5. In this way we get the limit value of Δf at each μ. Denoting $\alpha = \frac{G_M}{G_N}$, where G_N is the gravitational constant, we use the limit values of $(\mu[\text{GeV}], \alpha)$ for the attractive force from ref. [13]: for example, $(2.0 \times 10^{-22}, 3.0 \times 10^{-6})$, $(2.0 \times 10^{-20}, 1.6 \times 10^{-4})$,

$(2.0 \times 10^{-18}, 5.0 \times 10^{-4})$, and so on. Similarly, for the repulsive force the restrictions are found in ref.[14].

Let us consider two versions of Δm^2. First, if ν_τ is regarded as a candidate of dark matter, then Δm^2 is expected to be about 100eV^2 [15]. Second, according to Kamiokande atmospheric neutrino data [16], $\Delta m^2 \simeq 10^{-2}\text{eV}^2$. Next, we take as the energy of neutrino E the following three typical examples:

(i) KEK $\rightarrow$ Kamioka ($E = 1.4\text{GeV}$) [17]

(ii) FNAL $\rightarrow$ SOUDAN2 ($E = 10\text{GeV}$) [10]

(iii) $E = 1\text{TeV}$ (such neutrinos are detectable in e.g.DUMAND [18])

Figs.2a and 2b correspond to $\Delta m^2 = 100\text{eV}^2$ and $\Delta m^2 = 10^{-2}\text{eV}^2$ in the case that moduli interaction is an attractive force. The observable region is the upper part of the dotted line in (i), the dashed line in (ii) and the solid line in (iii), respectively. Namely the lines show the limit $\Delta f\phi = 10^{-3}\frac{\Delta m^2}{2E}$. As the energy of the neutrino increases, the detectable region becomes wide. The effect of moduli interaction is more significant in the case of Fig.2b than that of Fig.2a. Similarly in the case of repulsive force, the observable region of Δf is shown in Fig.3a ($\Delta m^2 = 100\text{eV}^2$) and in Fig.3b ($\Delta m^2 = 10^{-2}\text{eV}^2$).

We will comment on eq.(11) a little more. The formula in the brace can be written as

$$\frac{L}{c}\left(\frac{\Delta m^2}{4E} - \frac{\Delta f}{2}\phi\right), \tag{12}$$

for $f_{12} = 0$. In eq.(12) the first term is

$$\frac{L}{c}\frac{\Delta m^2}{4E} = 1.27\frac{\left(\frac{\Delta m^2}{\text{eV}^2}\right)}{\left(\frac{E}{\text{GeV}}\right)}\left(\frac{L}{\text{km}}\right), \tag{13}$$

and the second term is approximately given from eq.(6) as

$$
\begin{aligned}
\frac{\Delta f}{2}\phi &= \frac{3}{4}\frac{L}{c}\Delta f\frac{G_M M L}{\mu^3 R_E^3} \\
&= 2.54\Delta f\left(\frac{\alpha}{10^{-4}}\right)\frac{1}{\left(\frac{\mu}{10^{-20}\text{GeV}}\right)^2}\left(\frac{E}{\text{GeV}}\right)\left(\frac{L}{\text{km}}\right). \tag{14}
\end{aligned}
$$

When $\Delta f = 1$ and all other physical quantities are O(1) in the denoted units, both values of eqs.(13) and (14) are near $\frac{\pi}{2}$ which gives the maximum value of $P(\nu_\mu \rightarrow \nu_\tau)$.

We touch upon the effect of moduli interaction to solar neutrino oscillations briefly. Taking MSW effect [19][20] into account and using the number

density of electrons N_e [21] which reads

$$N_e(R) = 245 N_A \exp\left(-10.54 \frac{R}{R_{sun}}\right) \tag{15}$$

We get the probability that ν_e changes into ν_μ as

$$
\begin{aligned}
P(\nu_e \to \nu_\mu) &= \sin^2 \zeta_m \sin^2\left[\frac{1}{2c}\left\{\frac{\Delta m'^2}{2E}L_{sun} - \Delta f \Phi\right\}\right], &(16)\\
\Phi &= \frac{93\pi G_M}{\mu^2} E m_p \left(\alpha_p + \alpha_n + \frac{m_e}{m_p}\right) N_A R_{sun}\\
&\quad \times \left[\exp\left(-10.54\frac{R_{min}}{R_{sun}}\right) - \exp(-10.54)\right], &(17)
\end{aligned}
$$

where $\Delta m'^2 = m_2'^2 - m_1'^2$, $\Delta f = f_{22} - f_{11}$, α_p and α_n correspond to contribution of protons and neutrons, respectively. ν_e's are supposed to be generated around the distance of $R_{min}(\simeq 0.1 R_{sun})$ from the center and $L_{sun} = R_{sun} - R_{min} \simeq 0.9 R_{sun}$. Φ is the integrated value of ϕ with respect to R from R_{min} to R_{sun}. The second term inside the brace of eq.(16) is the effect of moduli interaction. The ratio of $|\Delta f \Phi|$ to $\Delta m'^2 L_{sun}/2E$ is estimated to be 8×10^{-9} for a typical example of $\mu = 10^{-22}$GeV, $G_M = 10^{-6}G_N$, $E = 10$eV and $\Delta m'^2 = 10^{-5}$eV^2. Thus we conclude that the moduli interaction affords no significant effect in the solar neutrino oscillation.

There are discussions on direct phenomenological consequences of moduli in cosmology in ref.[7] and others. Here we point out that moduli may give signals even in accelerator experiments, that is, long baseline neutrino oscillations. In some cases, the effect might be seen in a short baseline neutrino oscillation. For example, in the CHORUS experiment [22] they try to detect τ after ν_τ-nucleon charge current interactions. At $E \simeq 30$GeV, $L = 0.8$km, $\Delta f \phi/2 = 61\Delta f$ for $\mu \sim 10^{-20} - 10^{-24}$GeV. Then the experiment proves affirmative for moduli, if $|\Delta f| \gtrsim 10^{-2}$.

In conclusion if neutrino oscillation experiments will be scrupulously performed with various conditions, a clue of the form of moduli interaction with matter and mass of a modulus might be obtained. When moduli effect is detected in neutrino oscillation, there must exist at least one modulus with tiny mass and the structure of the true vacuum or the mechanism of SUSY breaking would be restricted.

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Figure captions

Fig.1: Schematic of Long Baseline Neutrino Oscillations.
The neutrino beam is injected from the accelerator at the point x_1 and detected at the point x_2.

Figs.2a, 2b: Observable region in the $(\mu, \Delta f)$ plane for the case of attractive force.
a: $\Delta m^2 = 100\mathrm{eV}^2$; b: $\Delta m^2 = 10^{-2}\mathrm{eV}^2$.
The observable regions are shown by the upper part of lines. The dotted, the dashed and the solid lines correspond to (i)$E = 1.4\mathrm{GeV}$, (ii)$10\mathrm{GeV}$, (iii)$1\mathrm{TeV}$, respectively.

Figs.3a, 3b: Observable region in the $(\mu, \Delta f)$ plane for the case of repulsive force.
The observable regions and other notations are the same as Fig.2.

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